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USE OF NON-LINEAR LOCALIZATION FOR ISOLATING STRUCTURES FROM EARTHQUAKE-INDUCED MOTIONS

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SUMMARY

The dynamic response due to earthquake-induced excitations of multi-storey buildings simulated by a cantilever (with attached concentrated masses) supported on a flexible foundation, is reconsidered when stiffness non-linearities are included. To this end, a suitable non-linear spring-mass device is placed between the ground and the mass of the foundation, which under certain conditions can absorb a significant amount of seismic energy over a large frequency range, thus drastically reducing the seismic response of the foundation. This is achieved by the stiffness non-linearity that gives rise to a localization phenomenon, according to which motions generated by external disturbances remain passively localized close to the point of seismic excitation instead of 'spreading' to the entire structure. The implications of these findings to the design of earthquake-resistant structures are discusssed. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: non-linear localization; base isolation; non-linear modes

1. INTRODUCTION

In the last decades there has been a steadily increasing interest towards the development of new techniques for isolating or reducing earthquake-induced structural responses. Popular methods rely on the absorption of earthquake energy by active or passive dampers and on base isolation of structures (cf. References 1–5). The purpose of this work is not to review the present state-of-the-art in this area, but rather to present a new way for base-isolating structures from earthquake-induced dynamical loads, by inducing non-linear localization phenomena.

The dynamic response of modern high-rise buildings is often determined through the analysis of an equivalent model of finite degrees of freedom, made from a massless cantilever, carrying concentrated masses at the floor levels. Multi-storey steel buildings may also be combined with a reinforced concrete-core resisting the wind and earthquake forces. In this case the effect of mass distribution of the cantilever should be included. Since studying a fully rigid steel support is rather

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unrealistic, the analysis considers a partially fixed cantilever with a translational and rotational spring allowing horizontal movement and rocking of the mass of the foundation. The effect of an initial constant axial compressive force at the tip of the cantilever (resulting for example due to a liquid tank) can be also taken into account. An exact dynamic analysis of the above model in which the cantilever column is treated as a continuous system and each concentrated mass has two degrees of freedom (due to translational and rotational motion) is presented by Kounadis.[6,7] The free and forced motions due to lateral dynamic forces or to ground motion induced by earthquakes is conventiently established in these references using generalized functions.

The present study deals with the above type of model of multi-storey buildings, and focuses on ground motion due to strong earthquakes. Quite often an unsatisfactory earthquake performance of such buildings is attributed to unsatisfactory seismic analysis (e.g. due to underestimation of the ductility, of the strength required at joints or of the significance of asymmetries). An efficient structural solution (being rather expensive) to overcome such uncertainties is to perform base isolation by absorbing seismic energy through appropriate devices at the foundation level. This work is based on a *non-linear* dynamic analysis, aiming at the introduction of a suitable secondary *non-linear mechanism* which will absorb a significant amount of seismic energy, leaving the foundation of the cantilever-column model with very small oscillation amplitude. In this way, the absorption of energy is not realized by any damper device as usually occurs in traditional earthquake isolation designs.

The idea of using non-linearities to induce motion confinement and energy localization in vibrating systems was developed in recent works,[8–10] where it was shown that *non-linear systems can be designed to possess localized responses and to confine passively vibrational energy*. These localization phenomena are due to the existence of localized non-linear normal modes (NNMs) which are non-linear analogues of the normal modes of classical linear vibration theory. Previous studies have shown that suitable placement of non-linear elements in a structure can alter its modal properties, introducing new stable modes that are localized in space and resulting in improved shock and vibration performance. In such designs, motions generated by external disturbances remain practically localized close to the point of their generation instead of 'spreading' to the entire structure. Previous works applied this non-linear localization design to mechanical systems.[8] It is the purpose of the present work to show that this non-linear localization phenomenon can be successfully employed to isolate structures from earthquake-induced disturbances.

2. PROBLEM DESCRIPTION

The model of the non-linear base-isolated structure is depicted in Figure 1. The *primary structure* to be isolated consists of a slender cantilever beam with uniform mass distribution, m, subjected to a constant compressive load P at its tip. The cantilever column carries n concentrated masses $M_1, M_2, \ldots, M_n$ (with corresponding rotational inertias $J_1, J_2, \ldots, J_n$) attached at points located at $x = a_1, a_2, \ldots, a_n$, with $a_n = l$, respectively, from the support. The cantilever column is partially fixed (with rotational spring stiffness c_3) to the mass M_0 of the foundation which is connected via a linear translational spring with stiffness c_2 to another small mass m_1 connected to the ground with a non-linear translational spring having linear and non-linear stiffness coefficients c_1 and γ, respectively. The mass m_1 with the non-linear stiffness is designated as the *secondary structure* of the system, and represents the location where the earthquake energy will be

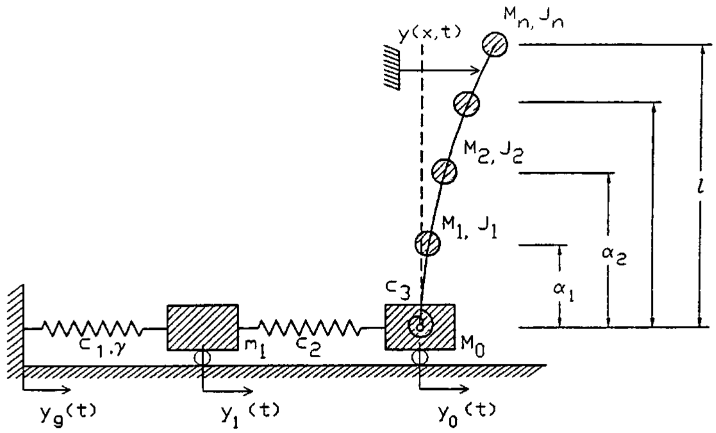

Figure 1

principally localized. *In essence, the proposed base isolation design amounts to the introduction of a secondary system acting like a non-linear vibration absorber* (*the secondary structure*), *capable of localizing seismic energy away from the primary structure over a wide range of frequencies. Once the seismic energy is localized then it can more effectively be eliminated by passive and/or active means.* The system of cantilever column and concentrated masses is subjected to a prescribed ground motion $y_g(t)$, representing the earthquake input. Note that the mass m_1 of the connected substructure must the small compared to the mass M_0 of the foundation, since, otherwise, the entire system will have little practical validity. In the following analysis, the mass m_1 and the non-linear stiffness coefficient γ are the main control parameters for inducing the localization phenomenon in the system.

The governing equations of motion can be established in terms of the displacements $y_1(t)$, $y_0(t)$ and $y(x, t)$ [the absolute displacement of the cantilever column] and their temporal and spatial derivatives by employing Hamilton's stationary-value variational principle, i.e. requiring that,

$$\delta \int_{t_1}^{t_2} (K_T - V_T)\, dt = 0 \tag{1}$$

where δ is the variational operator, and K_T and V_T are the functionals of the total kinetic energy and total potential energy, respectively, which are given by

$$\begin{aligned} K_T &= \frac{1}{2}\int_0^l m\dot{y}^2\, dx + \frac{1}{2}m_1\dot{y}_1^2 + \frac{1}{2}M_0\dot{y}_0^2 + \frac{1}{2}M_n\dot{y}^2(l,t) + \frac{1}{2}J_n\dot{y}'^2(l,t) \\ &\quad + \frac{1}{2}\int_0^l \sum_{i=1}^{n-1} M_i\dot{y}_i'^2\delta(x-a_i)\, dx - \frac{1}{2}\int_0^l \sum_{i=1}^{n-1} J_i\dot{y}_i'^2\delta'(x-a_i)\, dx \\ V_T &= \frac{\mathrm{EI}}{2}\int_0^l y''^2\, dx - \frac{P}{2}\int_0^l y'^2\, dx + \frac{1}{2}c_1(y_1-y_g)^2 + \frac{1}{2}c_2(y_1-y_0)^2 \\ &\quad + \frac{1}{p+1}\gamma(y_1-y_g)^{p+1} + \frac{1}{2}c_3 y'^2(0,t) \end{aligned} \tag{2}$$

where primes denote differentiation with respect to x, whereas dots denote differentiation with respect to time t; δ and δ' are the Dirac delta function and its generalized derivative, while EI is the flexural rigidity of the cantilever column. Assuming that all stiffness coefficients are positive, the non-linear spring between the ground and the mass m_1 is of the hardening type with a linear part and a linear/non-linear component of degree $p \geqslant 1$. We also note that the analysis can readily include lateral dynamic forces (see work by Kounadis[6,7]).

Taking into account the compatibility condition,

$$y(0, t) = y_0(t) \tag{3}$$

equation (1) coupled with relations (2), leads to the following system of equations:

$$\mathrm{EI}y''''(x, t) + Py''(x, t) + \left[m + \sum_{i=1}^{n-1} M_i \delta(x - a_i) \right] \ddot{y} - \sum_{i=1}^{n-1} J'_i \ddot{y}' \delta(x - a_i) = 0 \tag{4}$$

$$\mathrm{EI}y'''(l, t) = M_n \ddot{y}(l, t) - Py'(l, t) \tag{5}$$

$$\mathrm{EI}y''(l, t) = -J_n \ddot{y}'(l, t) \tag{6}$$

$$\mathrm{EI}y'''(0, t) = -c_2[y_0(t) - y_1(t)] - Py'(0, t) - M_0 \ddot{y}_0(t) \tag{7}$$

$$\mathrm{EI}y''(0, t) = c_3 y'(0, t) \tag{8}$$

$$m_1 \ddot{y}_1(t) + c_1[y_1(t) - y_g(t)] + \gamma[y_1(t) - y_g(t)]^p + c_2[y_1(t) - y_0(t)] = 0 \tag{9}$$

For the sake of simplicity we assume at this point that the cantilever possesses a single concentrated mass, i.e. $n = 1$; then, the above equations of motion and the boundary conditions take the following form:

$$\mathrm{EI}y''''(x, t) + \mathrm{P}y''(x, t) + m\ddot{y}(x, t) = 0 \tag{10}$$

$$\left.\begin{aligned} \mathrm{EI}y'''(l, t) &= M_1 \ddot{y}(l, t) - \mathrm{P}y'(l, t) \\ \mathrm{EI}y''(l, t) &= -J_1 \ddot{y}'(l, t) \\ \mathrm{EI}y'''(0, t) &= -c_2[y_0(t) - y_1(t)] - Py'(0, t) - M_0 \ddot{y}_0(t) \\ \mathrm{EI}y''(0, t) &= c_3 y'(0, t) \end{aligned}\right\} \tag{11}$$

$$m_1 \ddot{y}_1(t) + c_1[y_1(t) - y_g(t)] + \gamma[y_1(t) - y_g(t)]^p + c_2[y_1(t) - y_0(t)] = 0 \tag{12}$$

In the following section we study the above set of equations by discretization, employing the complete orthogonal set of eigenfunctions of the corresponding linear cantilever-foundation subsystem.

3. ANALYSIS

Equations (10) and (11) form a system of linear partial differential equations with linear boundary conditions. However, this system depends also on the variable $y_1(t)$ which is governed by the *non-linear* ordinary differential equation (12). Equations (10)–(12) are solved by employing previous results derived by Kounadis.[6,7] To this end, we consider equation (10) and boundary

conditions (11), and regard the dependent variable $y_1(t)$ as a *pseudo-forcing function*. Extracting the term containing $y_1(t)$ from the second of the boundary conditions (11) and placing it on the right-hand side (RHS) of the governing partial differential equation (10), and problem can be rewritten as follows:

$$\mathrm{EI}y''''(x,t) + Py''(x,t) + m\ddot{y}(x,t) = c_2 y_1(t)\delta(x) \tag{13}$$

$$\left.\begin{aligned}
\mathrm{EI}y'''(l,t) &= M_1\ddot{y}(l,t) - Py'(l,t)\\
\mathrm{EI}y''(l,t) &= -J_1\ddot{y}(l,t)\\
\mathrm{EI}y'''(0,t) &= -c_2 y(0,t) - Py'(0,t) - M_0\ddot{y}(0,t)\\
\mathrm{EI}y''(0,t) &= c_3 y'(0,t)
\end{aligned}\right\} \tag{14}$$

where relation (3) was taken into account.

The eigenvalue-problem associated with equations (13) and (14) and $y_1(t) = 0$ was solved exactly by Kounadis[6,7] using the Laplace transform technique. Hence, we can quote directly the results of that previous work in order to discretize the partial differential equation (13). This is accomplished by expressing the transverse cantilever-column displacement in the following series form:

$$y(x,t) = \sum_{i=1}^{\infty} \bar{Y}_n(x) A_n(t) \tag{15}$$

where $A_n(t)$ is the nth modal amplitude and $\bar{Y}_n(x)$ the corresponding normalized eigenfunction of the problem (13)–(14) with $y_1(t) = 0$. Since the later problem is self-adjoint the set of eigenfunctions considered is orthogonal and complete. The exact expression of $\bar{Y}_n(x)$ can be found in the appendix.

Substituting equation (15) into equations (13) and (14), taking into account the fact that by construction the eigenfunctions satisfy exactly all the boundary conditions, and making use of their orthogonality properties, the sets (13) and (14) are replaced by a set of discretized ordinary differential equations governing the modal amplitudes $A_n(t)$. Retaining only the two leading terms in series (15), the discretized equations are written in the following non-dimensional form:

$$\ddot{A}_i(\tau) + k_i^4 A_i(\tau) = \frac{\bar{c}_2 Y_i(0)}{\int_0^1 Y_i^2(\xi)\,\mathrm{d}\xi + \bar{M}_1 Y_i^2(1) + \bar{J}_1 Y_i'^2(1) + \bar{M}_0 Y_i^2(0)}\, y_1(\tau) \quad (i = 1, 2) \tag{16}$$

where the following non-dimensionalizations are introduced:

$$\xi = x/l,\ Y_i(\xi) = \bar{Y}_i(x)/l,\ k_i^4 = \frac{m\omega_i^2 l^4}{\mathrm{EI}},\ \tau = t\sqrt{\frac{\mathrm{EI}}{ml^4}}$$

$$\bar{M}_1 = \frac{M_1}{ml},\ \bar{J}_1 = \frac{J_1}{ml^3},\ \bar{c}_2 = \frac{c_2 l^3}{\mathrm{EI}}$$

In the above relations, ω_i^2 is the natural frequency squared and Y_i the non-dimensionalized eigenfunction of the ith mode. Complementing the above equations is the non-linear ordinary

differential equation (12) governing $y_1(t)$, which in view of equation (15), is expressed as follows:

$$\bar{m}_1 \ddot{y}_1(\tau) + \bar{c}_1[y_1(\tau) - y_g(\tau)] + \bar{\gamma}[y_1(\tau) - y_g(\tau)]^p + \bar{c}_2[y_1(\tau) - Y_1(0)A_1(\tau) - Y_2(0)A_2(\tau)] = 0 \tag{17}$$

where the new non-dimensional parameters are defined by

$$\bar{m}_1 = \frac{m_1}{ml}, \bar{\gamma} = \frac{\gamma l^{p+2}}{\mathrm{EI}}, \bar{c}_1 = \frac{c_1 l^3}{\mathrm{EI}}$$

and $y_1(t)$, $y(x, t)$ are rescaled according to, $y_1(t) \to l y_1(\tau)$, $y(x, t) \to l y(\xi, \tau)$.

By suitably selecting the system parameters, one can introduce a 1:1 internal resonance between the secondary substructure and any of the modes of the foundation-cantilever primary structure. Since such an internal resonance is most effective when low-order modes are involved, in the following numerical integrations the second flexural mode of the primary structure was selected to resonate with the secondary system. In such a case and for sufficiently small coupling between primary and secondary systems, non-linear mode localization occurs [8], whereby a stable localized mode exists with most of its energy spatially confined to the secondary substructure. This non-linear normal mode has no counterpart in linear theory and is solely due to the stiffness non-linearity of the secondary substructure; moreover, it exists over a wide range of frequencies of oscillation. When the overall system is forced by the earthquake excitation $y_g(\tau)$, most of the seismic energy is transferred to the localized mode and relatively small portions 'spread' to other (non-localized) modes of the structure. As a result, in the non-linear structure most of the seismic energy becomes passively confined in the secondary system and away from the structure to be isolated. More discussions of this non-linear localization phenomenon along with numerical integrations confirming it are presented in the following section.

4. NUMERICAL SIMULATIONS

In the numerical simulations presented in this section, the discretized linear equations (16) and the non-linear equation (17) were numerically integrated employing a fourth-order Runge–Kutta algorithm. In the first set of simulations the ground motion $y_g(t)$ was taken as a step function of finite or infinite time duration (see Figure 2), and the initial conditions were assumed to be equal to zero.

At this point the following additional non-dimensionalizations are introduced:

$$\beta^2 = \frac{Pl^2}{\mathrm{EI}}, \bar{M}_0 = \frac{M_0}{ml}, \bar{c}_3 = \frac{c_3 l}{\mathrm{EI}}, M_0^* = k^4 \bar{M}_0, M_1^* = k^4 \bar{M}_1, J_1^* = k^4 \bar{J}_1$$

These variables are used for computing the eigenfunctions of the system (see the appendix). The following numerical values are assigned to the various mass and stiffness parameters:

$$\bar{c}_2 = 0{\cdot}1, \bar{m}_1 = 10, \bar{c}_3 = 2, \bar{\gamma} = 4{\cdot}31, \bar{c}_1 = 0, \frac{k_i^4}{\omega_i^2} = \frac{ml^4}{\mathrm{EI}} = 0{\cdot}0431 \text{ (s/rad)}^2$$

$$\bar{M}_0 = 1, \bar{M}_1 = 1, \bar{J}_1 = 1, \beta^2 = \pi^2/40$$

$$k_1 = 0{\cdot}4245 \text{ (mode 1)}, \quad k_2 = 0{\cdot}8096 \text{ (mode 2)}$$

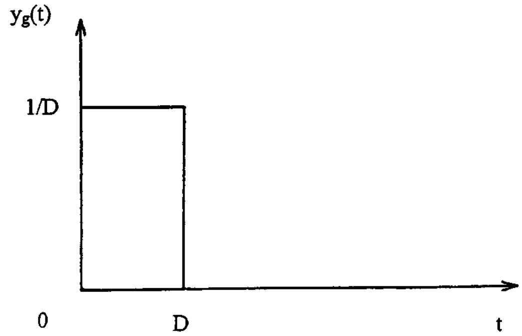

Figure 2

The degree of the non-linearity of the grounding stiffnes is taken either as $p = 1$ or 3, in order to compare the linear and non-linear responses of the system.

Considering first the linear system ($p = 1$), we note that when the stiffness c_2 tends to zero, the structure under consideration is partitioned into two parts; one part consists of the spring-mass system (m_1, γ); and the other of the cantilever column with its foundation mass M_0. We also note that in the linear case, the natural frequency of the spring-mass subsystem, $\Omega_1 = (\gamma/m_1)^{1/2} = 3{\cdot}1623$ rad/s is close to the natural frequency of the second mode of the other subsystem, $\omega_2 = 3{\cdot}157$ rad/s. As a result, when weak coupling between the two subsystems is introduced (i.e. $0 < c_2 \ll 1$), we expect a beat phenomenon to occur, whereby, energy from the directly excited spring – main subsystem, transfers to the other subsystem. Under such conditions, it is clear that the energy generated by the ground motion gets transferred back and forth between the two subsystems of the structure.

This is shown in Figure 3, where the amplitudes $y_1(t)$, $A_1(t)$ and $A_2(t)$ are plotted vs. time. *We note clearly the beat phenomenon between $y_1(t)$ and $A_2(t)$* (as predicted), and the small amplitude $A_1(t)$ of the first cantilever mode of the system. The forcing function $y_g(t)$ employed was taken as a step function (see Figure 2) of duration $D = \pi/(2\Omega_1)$ and amplitude $(1/D)$. From Figure 3 *we conclude that in the linear system, energy transfer to the cantilever column subsystem occurs, and no shock isolation of this subsystem from the disturbance generated by the ground motion is possible.*

We now consider the corresponding *non-linear* system with hardening cubic grounding stiffness ($p = 3$) under the action of identical ground motion $y_g(t)$. In previous studies[8–10] it was shown that non-linear systems with internal resonances, containing two weakly coupled subsystems can possess localized non-linear normal modes (NNMs) of vibration; these are synchronous vibrations where all material points of the system vibrate in-unison and the total energy of the system is localized in space. When a system possessing localized NNMs is perturbed by an external disturbance, motion confinement of vibrational energy can occur.[8] This is precisely what happens in the system under consideration: as mentioned previously there exists a 1 : 1 internal resonance between the secondary system and the second flexural mode of the primary structure, as well as weak coupling between the primary and secondary structures; as a result, the combined system possesses a localized mode that is mainly confined to the secondary system. As a consequence of the existence of the localized mode, vibrational energy induced into the system due to the ground motion $y_g(\tau)$ mainly localizes to the directly forced spring-mass subsystem, and relatively small

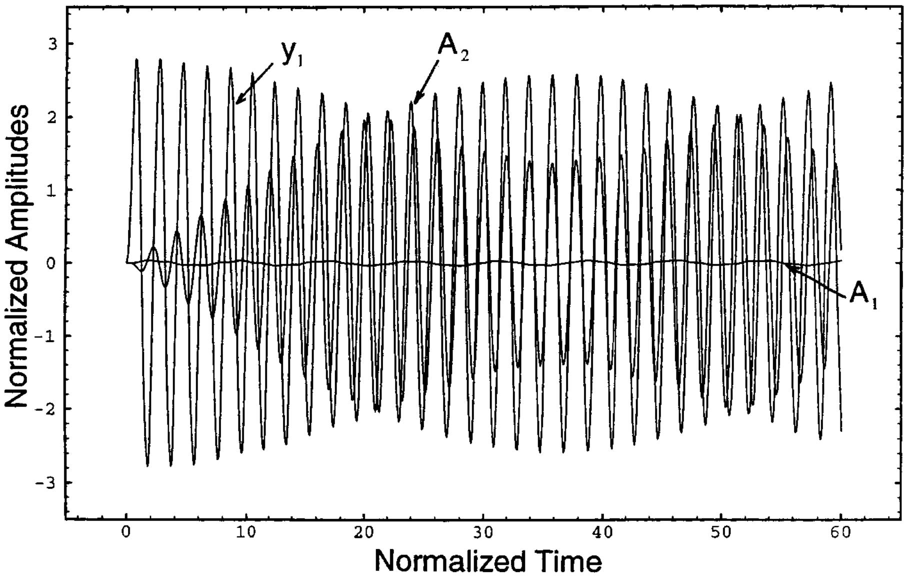

Figure 3

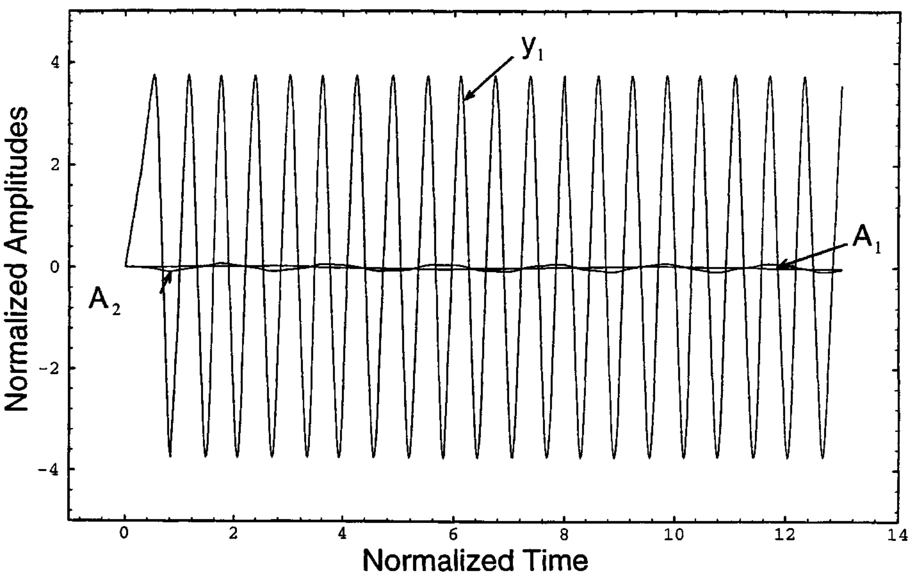

Figure 4

portions of this energy 'leak' to the cantilever substructure. This reduces significantly the portion of seismic energy transmitted to the primary structure.

This is presented in Figure 4, where the corresponding time histories of the co-ordinates of the system are depicted. *Note the nearly total localization that occurs in the co-ordinate* $y_1(t)$, *and the*

very small oscillation amplitudes of the modes of the cantilever column subsystem. This, inspite of the direct coupling between the two subsystems of the structure (since $c_2 \neq 0$). The reason for this passive motion confinement phenomenon lies in the structure of the NNMs of the unforced system (corresponding to $y_g = 0$). For weak stiffness c_2, the unforced system possesses a localized NNM whose energy is mainly confined to the mass m_1 (see Figure 1). Similar localized NNMs were detected in weakly coupled, undamped discrete or continuous oscillators with spatial periodicity.[8] When forcing is applied to the system (due to the ground motion), the resulting vibrational energy is directly transferred to the invariant manifold of the localized NNM where it remains for all time; as a result, the forced system appears to passively confine vibrational energy.

The effects of the size of the secondary mass m_1 compared to the mass of the foundation, and of the load duration on the results were studied in the numerical simulations depicted in Figure 5. As discussed previously, the small size of mass m_1 is essential for the proposed seismic design to be of any practical use. In Figure 5 the dynamic response of the non-linear system is depicted for $\bar{m}_1 = 0{\cdot}5$ and $\bar{\gamma} = 0{\cdot}2155$, and with all other parameters identical to the case depicted in Figure 4; ground motion $y_g(t)$ identical to the previous cases was considered. Although the mass m_1 is reduced considerably, the coefficient γ is also increased so that the frequency $\Omega_1 = (\gamma/m_1)^{1/2}$ of the corresponding linear system (with $p = 1$) remains equal to 3·1623 rad/s as in the previous cases. From Figure 5 we observe that *the non-linear localization phenomenon persists even for this reduced value of m_1*, and the only consequence in the dynamics is a larger amplitude modulation for $y_1(t)$. We conclude that *the non-linear motion confinement phenomena discussed herein occurs even in systems where the secondary mass m_1 is considerably smaller than the mass of the foundation, which is the primary structure that must be isolated*. This result demonstrates the design proposed is realistic and, thus, of practical importance.

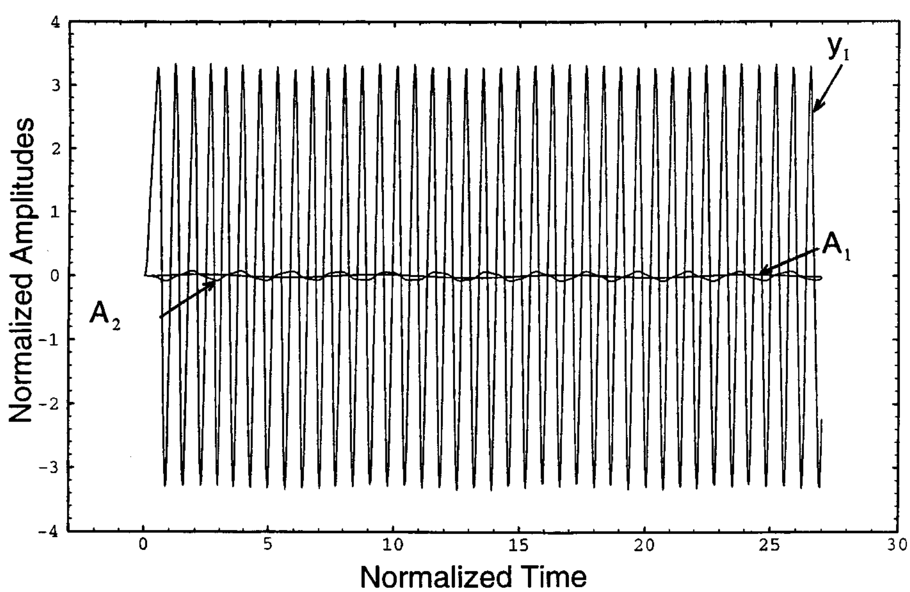

Figure 5

In an additional series of numerical simulations the effect of the time duration of the ground motion $y_g(t)$ on the dynamics was studied. The mass and stiffness parameters of the non-linear subsystem (with cubic nonlinearity, $p = 3$) were assigned the values $\bar{m}_1 = 0{\cdot}5$ and $\bar{\gamma} = 0{\cdot}2155$ while all other parameters were identical to the case depicted in Figure 4. The ground motion $y_g(t)$ was taken as a step function of unit amplitude and duration $D = 2\pi\Delta/\Omega_1$ with varying Δ. In Figure 6 the dynamic response of the system is plotted for $\Delta = 2$, 10 and ∞. Note that although

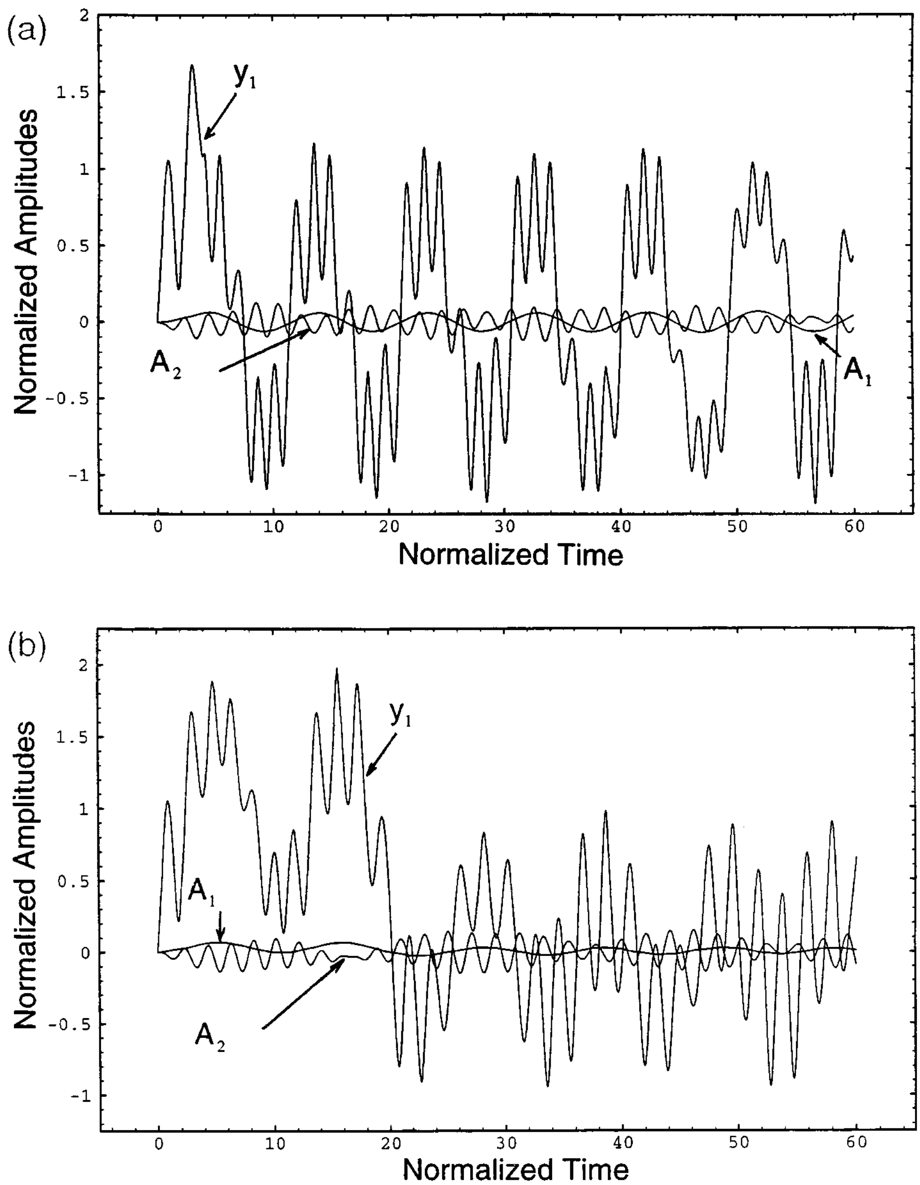

Figure 6

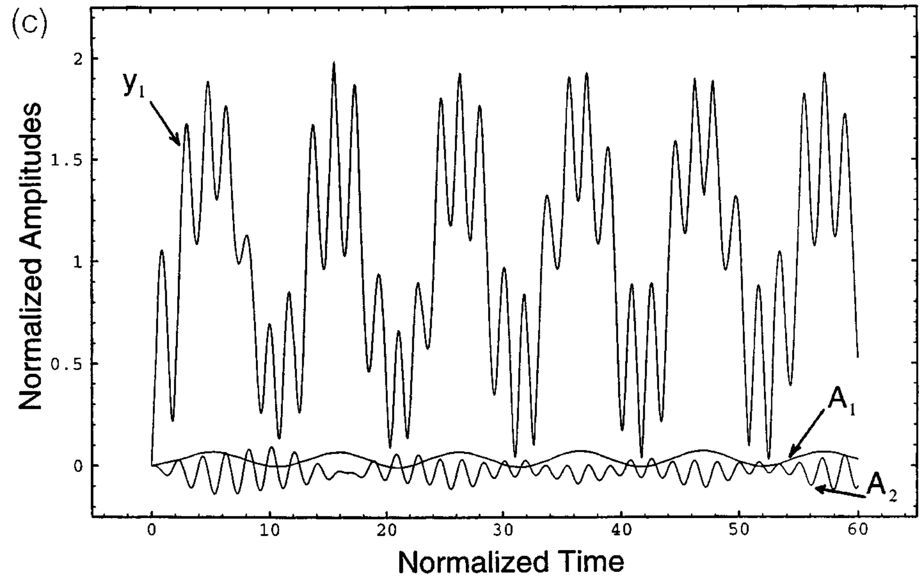

Figure 6. *Continued*

a relatively larger amount of energy gets transferred to the cantilever-column subsystem compared to the previous cases, *the principal amount of the seismic energy is still confined to the non-linear subsystem*; hence, the non-linear localization phenomenon still persists. Moreover, when the ground motion is a Heavy-side function ($\Delta = \infty$), the mean value of the amplitude $A_1(t)$ of the first mode of the cantilever column is positive, indicating a rigid body displacement of this subsystem (cf. Fig. 6c); this represents a rigid-body translational displacement of the cantilever column due to the step ground excitation, a result which agrees with physical intuition.

In a final series of numerical simulations, we considered the response of the system under a more realistic type of excitation. The ground motion chosen for this last simulation corresponds to a scaled El Centro Eathquake (cf. Figure 7). The input $y_g(t)$ was computed by numerically integrating twice the acceleration data. The parameters of the system were chosen to be identical to equation (18) and simulations of the dynamics with both linear and non-linear grounding stiffnesses were performed. For the non-linear case, we chose a grounding stiffness with clearance non-linearity, as depicted in Figure 8; the parameters for the clearance were chosen as, $\bar{\gamma}^* = \bar{\gamma} \times 10^5$ and $e = 1$ (cf. Figure 8). The non-linear localization of energy away from the primary structure can be clearly seen in the plots of Figure 9 where the linear and non-linear responses at various points of the system are superimposed for comparison purposes. Note that, due to the induced 1 : 1 internal resonance, nearly complete elimination of the response of the second mode of the primary structure occurs in the non-linear case, whereas the first modal amplitude of the same structure is also reduced. On the contrary, in the linear system a classic beat pheomenon occurs whereby energy from the secondary substructure is continuously exchanged with the second mode of the primary structure. This beat phenomenon prevents energy confinement in the linear case. Comparing the linear and non-linear responses of the primary structure, we note that the mere addition of clearance in the grounding stiffness results in an attenuation of vibration amplitude of the order of 60 per cent. This significant reduction in the amplitude is clearly

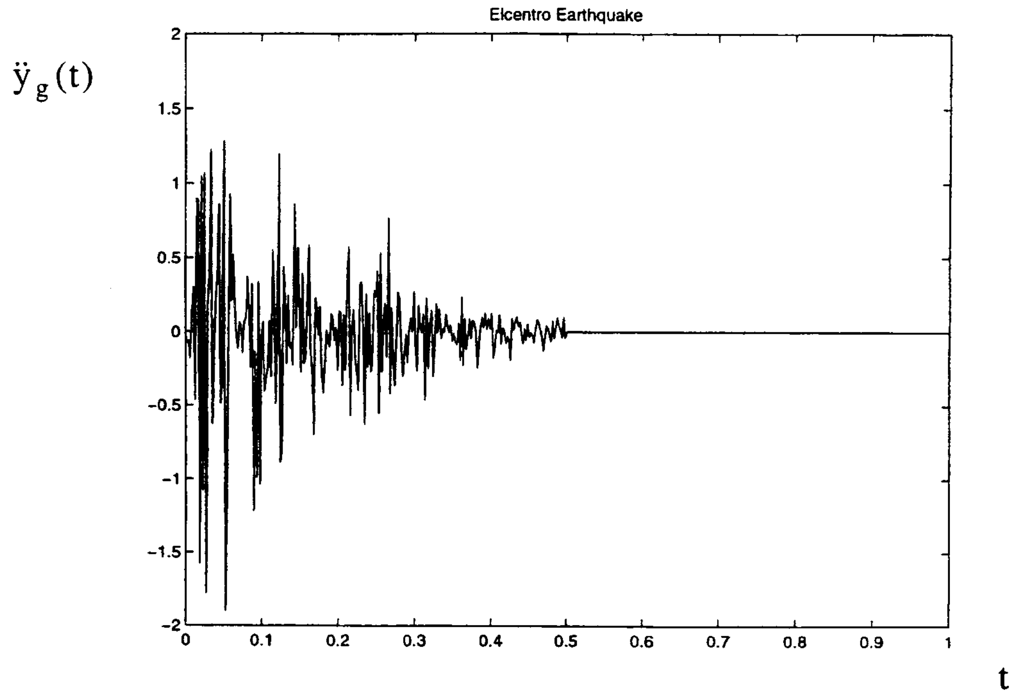

Figure 7

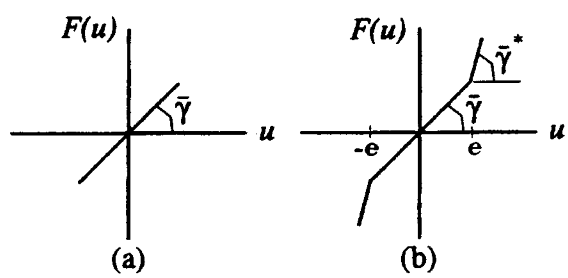

Figure 8

achieved due to the nearly complete elimination of the energy of the second mode of the primary structure by the non-linear localization phenomenon.

The implications of the presented dynamic phenomena to the isolation of the cantilever-column subsystem from ground motions should be apparent. In contrast to the linear case (see Figure 3), *in the non-linear system the cantilever-column subsystem is passively isolated from disturbances generated by the ground motion.* This isolation is solely due to the non-linearity of the grounding stiffness, and *does not require any damping or active control inputs.*

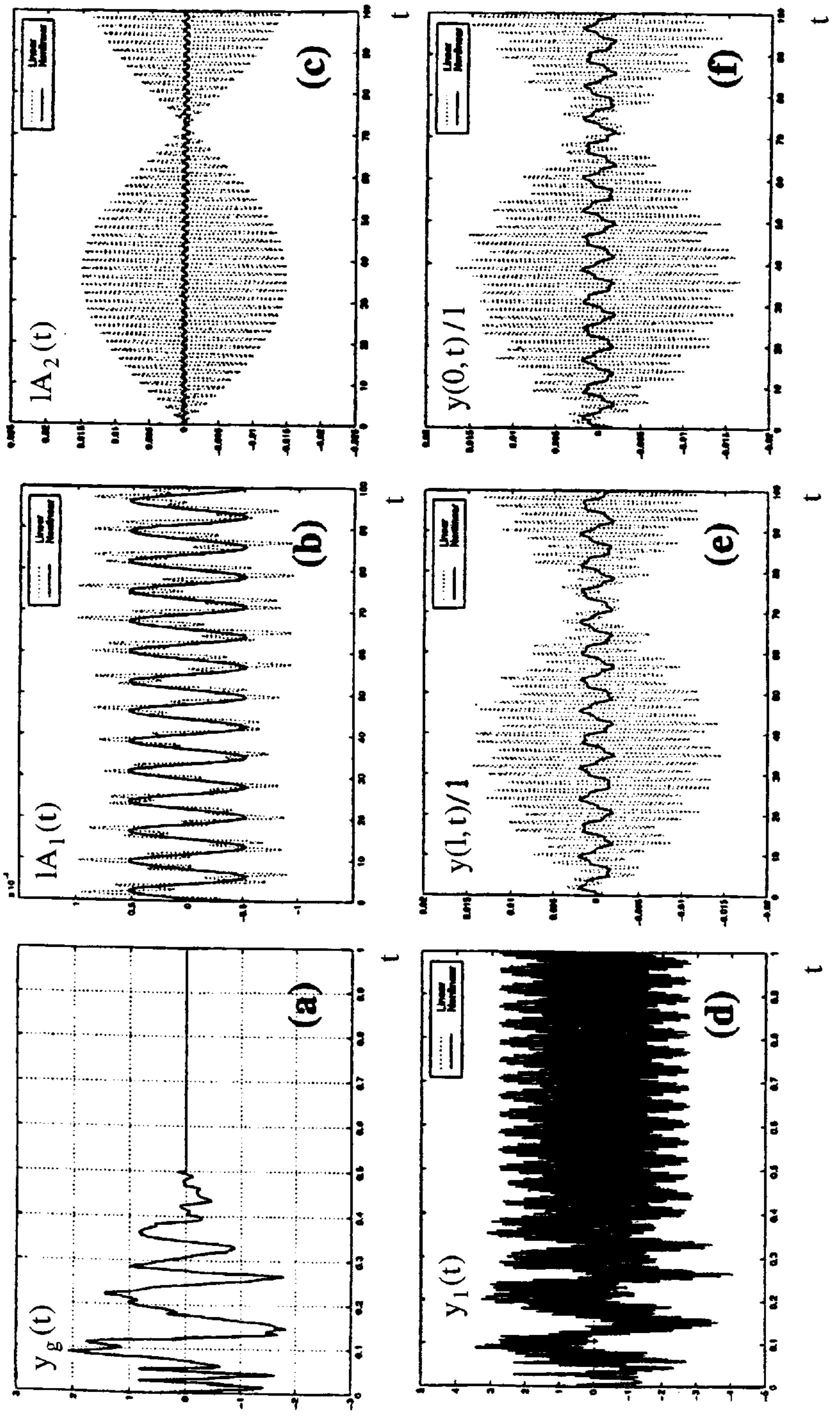

Figure 9

5. CONCLUSIONS

The application of the non-linear localization phenomenon to the isolation of a structure from vibrations induced by ground motions was throughly studied. It was shown that the addition of a single non-linear element to the structure can alter drastically its dynamics and induce passive motion confinement characteristics. Note that this element can absorb nearly all of the applied seismic energy, thus, restricting its transmission to the primary structure that must be isolated. The spring-mass device proposed responsible for the localization phenomenon, may be readily realized by using a small mass m_1 compared to the mass of the foundation, provided that the non-linear stiffness coefficient of the device is suitably chosen. Thus, such a design is realistic, and the ideas discussed in this work can be applied to the nonlinear design of structures with improved earthquake resistant capabilities. Moreover, the described non-linear localization phenomenon persists even when the rotational stiffness constant of the foundation is eliminated. The only effect on the dynamics of this stiffness elimination would be quantitative changes in the eigenvalues and eigenfunctions of the cantilever column subsystem.

APPENDIX
EIGENSOLUTIONS OF EQUATIONS (10) AND (11) WITH $y_1(t) = 0$[6,7]

The eigensolutions of the equations (10)–(11) are computed by seeking responses of the form,

$$y(x, t) = \bar{Y}(x)e^{j\omega t} \tag{18}$$

where $j = (-1)^{1/2}$. Introducing the non-dimensionalizations,

$$\xi = x/l,\ Y(\xi) = \bar{Y}(x)/l,\ k^4 = \frac{m\omega^2 l^4}{\mathrm{EI}},\ \beta^2 = \frac{Pl^2}{\mathrm{EI}},\ \bar{\beta}_0 = \frac{M_0}{ml},\ \bar{M}_1 = \frac{M_1}{ml}$$

$$\bar{J}_1 = \frac{J_1}{ml^3},\ \bar{c}_3 = \frac{c_3 l}{\mathrm{EI}},\ \bar{c}_2 = \frac{c_2 l^3}{\mathrm{EI}},\ M_0^* = k^4\bar{M}_0,\ M_1^* = k^4\bar{M}_1,\ J_1^* = k^4\bar{J}_1$$

substituting equation (18) into equations (10) and (11), and applying Laplace Transform with respect to $\hat{\imath}$, the normalized eigenfunction is expressed as

$$Y(\xi) = \varphi_1(\xi)Y(0) + \varphi_2(\xi)Y'(0) \tag{19}$$

where,

$$\varphi_1(\xi) = F_1(\xi) - (\bar{c}_2 - M_0^*)F_2(\xi)$$

$$\varphi_2(\xi) = F_2''(\xi) + \bar{c}_3 F_2'(\xi)$$

$$F_1(\xi) = \frac{1}{\varepsilon^2 + \zeta^2}(\varepsilon^2 \cosh \zeta\xi + \zeta^2 \cos \varepsilon\xi), \tag{20}$$

$$F_2(\xi) = \frac{1}{\varepsilon^2 + \zeta^2}\left(\frac{1}{\zeta}\sinh \zeta\xi - \frac{1}{\varepsilon}\sin \varepsilon\xi\right)$$

$$\zeta = \left\{-\frac{\beta^2}{2} + \left[\frac{\beta^4}{4} + k^4\right]^{1/2}\right\}^{1/2},\ \varepsilon = \left\{\frac{\beta^2}{2} + \left[\frac{\beta^4}{4} + k^4\right]^{1/2}\right\}^{1/2}$$

The constant coefficients $Y(0)$ and $Y'(0)$ in equation (19) are computed by solving the following set of homogeneous algebraic equations:

$$\begin{bmatrix} f_1(1) & f_2(1) \\ f_3(1) & f_4(1) \end{bmatrix} \begin{Bmatrix} Y(0) \\ Y'(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{21}$$

where

$$\left.\begin{aligned} &f_1(1) = \varphi_1''(1) - J_1^* \varphi_1'(1), f_2(1) = \varphi_2''(1) - J_1^* \varphi_2'(1) \\ &f_3(1) = \varphi_1'''(1) + M_1^* \varphi_1(1) + \beta^2 \varphi_1'(1), \\ &f_4(1) = \varphi_2'''(1) + M_1^* \varphi_2(1) + \beta^2 \varphi_2'(1) \end{aligned}\right\} \tag{22}$$

For non-trivial solutions in equation (21) we require that the determinant of co-efficients be equal to zero. This condition leads to the following algebraic equation that determines the countable infinity of normalized eigenfrequencies k_i of the system

$$R(k^4) = f_1(1) f_4(1) - f_3(1) f_2(1) = 0 \tag{23}$$

Setting $Y'(0) = 1$, for a specific normalized eigenfrequency k_i the coefficient $Y(0)$ is determined by either one of the homogeneous equations (21). This completes the calculation of the normalized eigenfunction $Y_i(\hat{t})$.

Taking into account the previous non-dimensionalisations, the ith eigenfunction is computed by the relation

$$\bar{Y}_i(x) = l Y_i(\xi) \tag{24}$$

and the ith eigenfrequency by

$$\omega_i = k_i^2 \sqrt{\frac{\mathrm{EI}}{ml^4}} \tag{25}$$

In Table I, we present the solutions for the non-dimensionalized eigenfrequencies k_i and the co-efficients $Y(0)$ for the first two modes and for varying values of $\bar{c}_2$. The following values for the

Table I. Numerical values of the parameters of the eigen-solutions of the cantilever-column subsystem ($Y'(0) = 1$)

	k_i		$Y(0)$	
$\bar{c}_2$	Mode 1	Mode 2	Mode 1	Mode 2
0	0	0·8049	∞	− 0·8893
0·01	0·2401	0·8053	317·455	− 0·8967
0·05	0·3582	0·8071	61·905	− 0·9272
0·10	0·4245	0·8096	29·972	− 0·9674
0·20	0·5011	0·8149	14·027	− 1·056
0·40	0·5857	0·8278	6·118	− 1·2645
0·80	0·6664	0·8625	2·358	− 1·8059
1·20	0·7014	0·9039	1·289	− 2·4565

parameters of the problem were assumed:

$$\beta^2 = \frac{\pi^2}{40}, \bar{M}_0 = 1, \bar{M}_1 = 1, \bar{J}_1 = 1, \bar{c}_3 = 2, Y'(0) = 1$$

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